ARTICLE

Total variation-based image inpainting and denoising using a primal-dual active set method

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This paper presents a formulation of an ℓ^2-based variational model implementable in simultaneous image inpainting and Gaussian denoising. To solve the optimization problem, an active set method that utilizes generalized derivatives on the regularized L^2 TV-type problem and its dual problem is applied. A numerical study ends the paper.

INTRODUCTION

Image inpainting involves the filling-in of removed, damaged, or unwanted regions in images. The inpainting is done usually by extending towards the inpainting region the information current on the boundaries of the region. This is synonymous with image interpolation wherein continuously defined data is constructed on a region in such a way that the region blends well with the surrounding features. Inpainting is widely practiced by artists in restoring artistic paintings.

Bertalmio and colleagues (2000) first applied inpainting to digital images by using high order PDE models. Since then, digital inpainting has spurred the research and development of various approaches including, e.g., variational techniques, wavelet-based methods, combination of wavelets and total variation minimization, elastica model, isotropic diffusion, etc. See Chan et al. (2002), Caselles et al. (1998), Chan et al. (2006), Oliveira et al. (2001) for examples. Recently, Cai et al. (2009) presented a split Bregman approach to solving a variation model for inpainting, while a median filter was introduced by Noori and Saryazdi (2010).

A popular variation model in reconstructing blocky images corrupted with Gaussian noise is the Rudin-Osher-Fatemi (ROF) model (Rudin et al. 1992)

$$\min_{u\in BV(\Omega)} \frac{1}{2}\int_\Omega (u-u_0)^2\,dx + \alpha\int_\Omega |\nabla u|\,dx \qquad (1)$$

where Ω is a simply connected bounded domain in $\mathbb{R}^2$ with Lipschitz continuous boundary $\partial\Omega$, and $\alpha > 0$ is the regularization parameter. The observed image is u_0 and $BV(\Omega)$ denotes the space of functions of bounded variation. A function $u \in L^1(\Omega)$ is said to be of bounded variation if the BV seminorm defined by

$$\int_\Omega |Du| = \sup\left\{\int_\Omega u \operatorname{div} v(x)dx : v \in \left(C_0^\infty(\Omega)\right)^2, |v(x)|_{\ell^\infty} \le 1, x \in \Omega\right\}$$

is finite (see Casas et al. 1999, Giusti et al. 1984 for details). The first integral term in (1) is the fidelity term that recovers essential features of the image. The second term is the total variation (TV) term that penalizes high oscillations associated with noise in the image but allows discontinuities associated with edges.

Chan and Shen (2001) proposed a restricted version of the ROF model for inpainting non-texture type images:

$$\min_{u\in BV(E\cup D)} \frac{1}{2}\int_E (u-u_0)^2\,dx + \alpha\int_{E\cup D} |\nabla u|\,dx \qquad (2)$$

In (2), E is any fixed closed domain outside the inpainting domain D, and $|\cdot|$ denotes the Euclidean norm. The image domain $E \cup D$ is usually taken to be a square. Since the TV model (2) is nondifferentiable, Chan and Shen introduced a global smoothing

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parameter to the TV term and used a low pass filter and a Gauss-Jordan iteration scheme to obtain a steady solution.

An elastic variational model for image reconstruction on a rectangular domain Ω is

$$\min_{u\in BV(\Omega)} \frac{1}{2}\int_{\Omega} |Ku-d|^2 + \alpha \int_{\Omega} |\nabla u| + \frac{\mu}{2}\int_{\Omega} |u|^2 \qquad (3)$$

where $K : L^2(\Omega) \to L^2(\Omega)$ is a continuous linear operator. For $\mu > 0$, problem (3) is guaranteed coercivity and uniqueness of solution (Chavent and Kunisch 1997). An augmented Lagrangian method based on a regularization of (3) and on its predual was developed for image denoising (see, Hintermüller and Kunisch (2004)). The method was shown to converge superlinearly and was demonstrated to be effective in removing Gaussian noise off images.

In this paper, we modify the discretized version of the variational model (3) to make it amenable to image inpainting and denoising, with K taken as the identity matrix. The primary change is in the restriction of the fidelity term to the non-inpainting domain E. We present a primal-dual framework by obtaining the Fenchel dual of the model and consequent optimality conditions. Neri and Zara (2012) made a similar approach with the regularizing term being $\int_\Omega |\nabla u|^2$ instead of $\int_\Omega |u|^2$. The former tends toward the banding effect associated with TV models due to a more pronounced diffusing process, whereas the latter aims for the minimal solution of the problem.

We apply an active-set approach to solve the resulting model. This approach is based on the approach taken by Hintermüller et al. (2003). Numerical results show that the method works fast and exhibits good denoising and inpainting capabilities on sufficiently thin domains.

A Variation Model

In the discrete setting, we concatenate the $n \times n$ image matrix u to an image vector $v \in \mathbb{R}^N$, $N = n^2$. Let

$$[\nabla v]_l = \left[(\nabla_x v)_l, (\nabla_y v)_{l+n}\right]^T, 1 \le l \le n,$$

where ∇_x and ∇_y are approximated by forward differences. The inpainting domain is D, and $E = \{i \in \{1, \ldots, N\} | i \notin D\}$. A discretized version of the total variation image inpainting model (2) is

$$\min_{v\in\mathbb{R}^N} \frac{1}{2}\|v - v^0\|^2_{\{E\}} + \alpha \sum_{l=1}^{N} \|[\nabla v]_l\| \qquad (4)$$

where v^0 is the observed image, $\|\cdot\|$ is the Euclidean norm and $\|\cdot\|_{\{W\}}$ is defined as

$$\|x\|_{\{W\}} = \sqrt{\sum_{i\in W} x_i^2}$$

A regularization of (4) is the following:

$$\min_{v\in\mathbb{R}^N} \left\{\frac{1}{2}\|v - v^0\|^2_{\{E\}} + \alpha \sum_{l=1}^{N} \|[\nabla v]_l\| + \frac{\mu}{2}\|v\|^2\right\} \qquad (\mathcal{P})$$

This is a discretized version of (3) where the fidelity term is restricted to E and $K = I$, identity. The ℓ^2-type problem $(\mathcal{P})$, although nondifferentiable, is strictly convex and is guaranteed a unique solution. When the inpainting region D is empty, $(\mathcal{P})$ reverts to the discrete form of model (3).

In order to construct a primal-dual active set method to inpainting, we use the Fenchel duality to find the dual of problem $(\mathcal{P})$ (see, Ekeland and Temam (1999) for details on the Fenchel dual). Let $F : \mathbb{R}^N \to \mathbb{R}$ and $G : \mathbb{R}^{2N} \to \mathbb{R}$ such that

$$F(v) = \Phi(v) + \frac{\mu}{2}\|v\|^2 \qquad (5)$$

and

$$G(w) = \sum_{l=1}^{N} \|[w]_l\| \qquad (6)$$

where

$$\Phi(v) = \begin{cases} \frac{1}{2}\|v - v^0\|^2_{\{E\}} & \text{on } E \\ 0 & \text{on } D \end{cases}$$

Note that in place of the fidelity term, the value of F over the inpainting domain D is equivalently zero.

The convex conjugate of (5), $F^*(v^*)$, is defined by

$$F^*(v^*) = \sup_{v\in\mathbb{R}^N} \{\langle v, v^*\rangle - F(v)\}$$

Let v_W be the components of v with indices in W. There are two cases for entries in the optimal solution $\tilde{v}$:

Case 1: For entry values over the index set E, we have

$$v^*_E - \bar{v}_E + v^0_E - \mu\bar{v}_E = 0$$
$$\bar{v}_E = (1+\mu)^{-1}(v^*_E + v^0_E)$$

Case 2: Over the set D, the fidelity term is zero and only the regularization term is present in (5). Thus,

$$\bar{v}_D = \mu^{-1} v^*_D$$

These two cases lead to the following result.

Theorem 1. *Given the regularized primal model ($\mathcal{P}_\gamma$). The convex conjugate of (5) is given by*

$$F^*(v^*) = \frac{1}{2}|||v^* + v^0|||^2_{\{E\}} - \frac{1}{2}\|v^0\|^2_{\{E\}} + \frac{\mu^{-1}}{2}|||v^*|||^2_{\{D\}} \quad (7)$$

where

$$|||x|||^2_{\{X\}} = \begin{cases} \langle (1+\mu)^{-1}x, x\rangle, & if \;\; X = E \\ \langle \mu^{-1}x, x\rangle, & if \;\; X = D. \end{cases}$$

It is shown in Vogel (2002) that the conjugate of G, $G^*(w^*)$, is given by

$$G^*(w^*) = \mathcal{I}_{\{W\}}(w^*) = \begin{cases} 0, & if \;\; w^* \in \mathcal{W} \\ +\infty, & if \;\; w^* \notin \mathcal{W} \end{cases}$$

where $\mathcal{I}$ is the indicator function, $W = \{w^* \in \mathbb{R}^{2N} \mid \|[w^*]_l\| \le \alpha, l = 1,2, \ldots, N\}$. Setting $p = -v^*$ and $\Lambda^* = \nabla^T = -\text{div}$, we have the following result:

Theorem 2. *The dual form of ($\mathcal{P}$) is given by*

$$\sup_{p\in\mathbb{R}^{2N}, \|[p]_l\|\le\alpha} \left\{-\frac{1}{2}|||\text{div}\, p + v^0|||^2_{\{E\}} + \frac{1}{2}\|v^0\|^2_{\{E\}} - \frac{\mu^{-1}}{2}|||\text{div}\, p|||^2_{\{D\}}\right\}. \quad (\mathcal{D})$$

Due to the non-uniqueness of the solution of the dual problem ($\mathcal{D}$), we add a Tikhonov-type of regularization similar to the method used by Hintermüller and Stadler (2006). This results in

$$\sup_{p\in\mathbb{R}^{2N}, \|[p]_l\|\le\alpha} \left\{-\frac{1}{2}|||\text{div}\, p + v^0|||^2_{\{E\}} + \frac{1}{2}\|v^0\|^2_{\{E\}} - \frac{1}{2}|||\text{div}\, p|||^2_{\{D\}} - \frac{\gamma}{2\alpha}\|p\|^2\right\} \quad (\mathcal{D}_\gamma)$$

where $\gamma > 0$. The constraint $\|[p]_l\| \le \alpha$ is expressed in the objective in penalty form as the indicator function $I_w(-p)$. We utilize the Fenchel duality theorem again to find the dual problem for ($\mathcal{D}_\gamma$). With $\Lambda^* = \text{div}$ and $q = -p$, let

$$\mathcal{F}^*_\gamma(q) = \frac{\gamma}{2\alpha}\|q\|^2 + I_W(q) \quad (8)$$

$$\mathcal{G}^*_\gamma(\Lambda q) = \frac{1}{2}|||v^0 - \Lambda q|||^2_{\{E\}} - \frac{1}{2}\|v^0\|^2_{\{E\}} + \frac{1}{2}|||\Lambda q|||^2_{\{D\}} \quad (9)$$

To obtain the dual $\mathcal{G}_\gamma$ of $\mathcal{G}^*_\gamma$, we consider two cases.

Case 1: Over the set E, we have

$$\mathcal{G}_\gamma(x) = \sup_{x^*\in\mathbb{R}^N} \left\{\langle x^*, x\rangle - \frac{1}{2}|||v^0 - x^*|||^2_{\{E\}} + \frac{1}{2}\|v^0\|^2_{\{E\}}\right\}$$

The solution $\bar{x}^*$ is given by

$$\bar{x}^* = (1+\mu)x + v^0$$

$\mathcal{G}_\gamma(x)$ over E becomes

$$\mathcal{G}_\gamma(x) = \frac{1}{2}\|x + v^0\|^2_{\{E\}} + \frac{\mu}{2}\|x\|^2_{\{E\}}$$

Case 2: Over the set D, we have

$$\mathcal{G}_\gamma(x) = \sup_{x^*\in\mathbb{R}^N} \left\{\langle x^*, x\rangle - \frac{1}{2}|||x^*|||^2_{\{D\}}\right\}$$

whose supremum is at $\bar{x}^* = \mu x$. Thus, on D we obtain

$$\mathcal{G}_\gamma(x) = \frac{\mu}{2}\|x\|^2_{\{D\}}$$

Combining these two cases, we have the following result.

Theorem 3. *Given the regularized model ($\mathcal{D}_\gamma$), the dual of $\mathcal{G}^*_\gamma(x^*)$ is given by*

$$\mathcal{G}_\gamma(x) = \frac{1}{2}\|x + v^0\|^2_{\{E\}} + \frac{\mu}{2}\|x\|^2. \quad (10)$$

On the other hand, the convex conjugate of $\mathcal{F}^*_\gamma(x)$ is

$$\mathcal{F}_\gamma(x) = \sum_{l=1}^{N} [\Psi(\nabla v)]_l$$

where

$$[\Psi(\nabla v)]_l = \begin{cases} \frac{\alpha}{2\gamma}\|[\nabla v]_l\|^2 & \text{if } \|[\nabla v]_l\| < \gamma \\ \alpha\left(\|[\nabla v]_l\| - \frac{\gamma}{2}\right) & \text{if } \|[\nabla v]_l\| \ge \gamma \end{cases}$$

for $l = 1,2,\ldots, N$. See, Dong et al. (2009) for the computations. Using Theorem 3 above, with $v = -x$ and $\Lambda = -\nabla$, we get the following result:

Theorem 4. *The dual form of the model ($\mathcal{D}_\gamma$) is given by*

$$\min_{v\in\mathbb{R}^N} \left\{\mathcal{F}_\gamma(\nabla v) + \frac{1}{2}\|v - v^0\|^2_{\{E\}} + \frac{\mu}{2}\|v\|^2\right\}. \quad (\mathcal{P}_\gamma)$$

It is shown in Hintermüller and Stadler (2006) that as $\gamma \to 0$, the solution of $\mathcal{P}_\gamma$ converges to that of $\mathcal{P}$. The solutions to the primal ($\mathcal{P}_\gamma$) and dual ($\mathcal{D}_\gamma$) problems, given by $\bar{v}_\gamma$ and $\bar{p}_\gamma$, respectively, satisfy the following optimality conditions (see, Ekeland and Temam (1999)):

$$\mu\bar{v}_\gamma + \bar{v}_\gamma - \text{div}\,\bar{p}_\gamma = v^0 \quad \text{on } E \quad (11)$$

$$\mu\bar{v}_\gamma = \text{div}\,\bar{p}_\gamma \quad \text{on D} \quad (12)$$

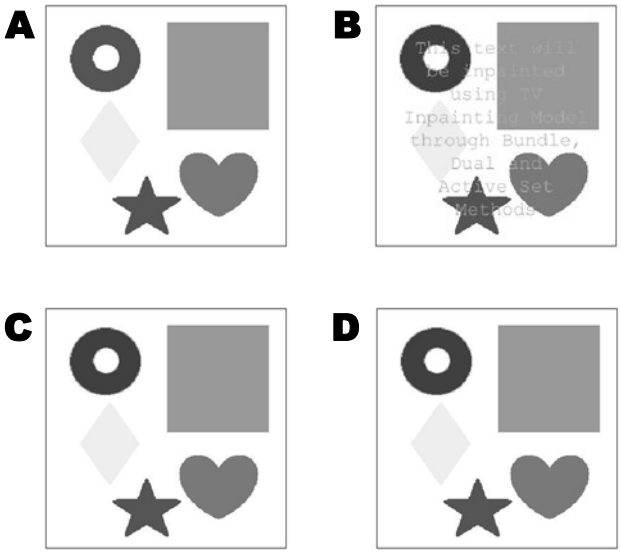

Figure 1. Test image 1. A, Original image; **B**, Image with text; **C**, PDAS_I; and **D**, Split Bregman method.

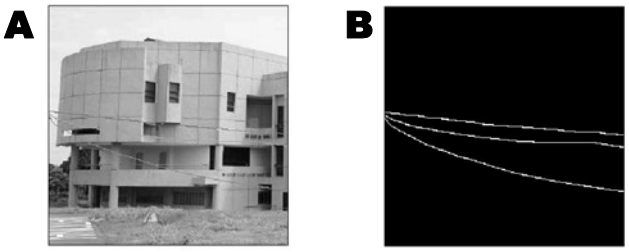

Figure 2. Test image 2. A, Original image; and **B**, Inpainting domain.

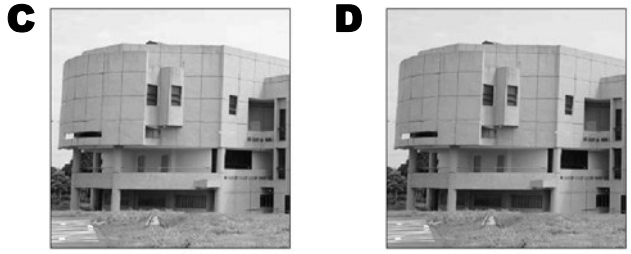

Figure 3. Image reconstructions. A, PDAS_I; and **B**, Split Bregman method.

$$\begin{cases} \gamma[\bar{p}_\gamma]_l - \alpha[\nabla\bar{v}_\gamma]_l = 0 & \text{if } \left\|[\bar{p}_\gamma]_l\right\| < \alpha \\ \left\|[\nabla\bar{v}_\gamma]_l\right\| [\bar{p}_\gamma]_l = \alpha[\nabla\bar{v}_\gamma]_l & \text{if } \left\|[\bar{p}_\gamma]_l\right\| = \alpha \end{cases} \quad \text{on} \quad E \cup D \quad (13)$$

for $l = 1,\ldots, N$. Let $\kappa \in \mathbb{R}^N$ with $\kappa_i = 1$ if pixel-index $i \in E$; 0 otherwise. We combine equations (11) and (12) as

$$\mu\bar{v}_\gamma - \operatorname{div}\bar{p}_\gamma + \kappa_E(\bar{v}_\gamma - v^0) = 0 \quad (14)$$

where $\kappa_E = D(\kappa)$, the $N \times N$ diagonal matrix. The optimal conditions in (13) can also be combined as:

$$\max\left(\gamma, \left\|[\nabla\bar{v}_\gamma]_l\right\|\right)[\bar{p}_\gamma]_l - \alpha\nabla[\bar{v}_\gamma]_l = 0 \quad (15)$$

for every $l = 1,2,\ldots, N$. In the next section, we present a Newton -type solution method based on the optimality conditions presented here.

An Active Set Method

Active set methods have become popular techniques in solving TV problems such as theROF. In Kärkkäinen et al. 2001, the authors proposed an active-set method based on augmented Lagrangian smoothing to solve TV optimization problems with L^2 data-fidelity term. The primal-dual variable updates in their algorithm were based on first order optimality conditions. When the variation problem is sufficiently smooth, an active set method can be developed that utilizes second-order information of the problem. In Hintermüller and Kunisch (2004), active set methods based on primal-dual structures were shown to be semismooth Newton methods which exhibit superlinear convergence. In Hintermüller and Stadler (2006), a convergent active set method on a regularized total variation problem for denoising was presented.

Utilizing equations (14) and (15), we develop a semismooth active set method in Hintermüller et al. (2003). The derivation scheme is similar to that done in Neri and Zara (2012). A Newton step to (11), (12), and (15) at the k-th approximations v^k and p^k can be obtained as follows:

$$\begin{pmatrix} \mu I_N + \kappa_E & -\operatorname{div} \\ G\nabla & D(m^k) \end{pmatrix}\begin{pmatrix} \delta_v \\ \delta_p \end{pmatrix} = \begin{pmatrix} -\mu v^k + \operatorname{div} p^k + \kappa_E(-v^k + v^0) \\ \alpha\nabla v^k - D(m^k)p^k \end{pmatrix} \quad (16)$$

where

$$G = -\alpha I_{2N} + \chi_{\mathcal{A}_{k+1}} D(p^k) J(\nabla v^k)$$

$$m^k = \max\left(\gamma I_{2N}, \eta(\nabla v^k)\right) \in \mathbb{R}^{2N}$$

with the mapping $\eta: \mathbb{R}^{2N} \to \mathbb{R}^{2N}$ given by

$$\left(\eta(v)\right)_i = \|v_i\| \text{ with } v \in \mathbb{R}^{2N}, i = 1, \ldots, 2N$$

The active set indicator $\chi_{\mathcal{A}_{k+1}} = D(t^k)$ is a $2N \times 2N$ diagonal matrix with

$$t_i^k := \begin{cases} 1 \text{ if } \left(\eta(\nabla v^k)\right)_i \geq \gamma \\ 0 \text{ if } \left(\eta(\nabla v^k)\right)_i < \gamma \end{cases}$$

That is, if the i^{th} component is part of the active set $\mathcal{A}_{k+1}$, then $t_i = 1$; else, it is 0. The Jacobian of η is

$$J(\nabla v) = \left(D\left(\eta(\nabla v)\right)\right)^{-1}\begin{pmatrix} D(\nabla_x v)\, D\left(\nabla_y v\right) \\ D(\nabla_x v)\, D\left(\nabla_y v\right) \end{pmatrix}$$

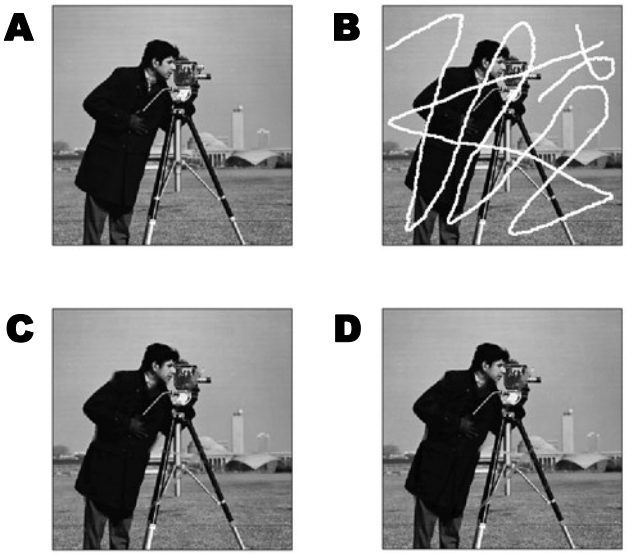

Figure 4. Test image 3. A, Original image; **B**, Degraded image; **C**, PDAS_I; and **D**, split Bregman method.

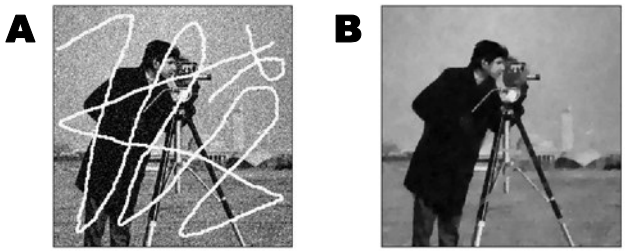

Figure 5. Inpainting and denoising. A, Noisy image; and **B**, PDAS_I

Since all components of m^k are positive, the diagonal matrix $D(m^k)$ is invertible. The steps δ_p and δ_v become

$$\delta_p = \alpha D^{-1}(m^k)\nabla v^k - p^k - D^{-1}(m^k)G\nabla\delta_v \qquad (17)$$

and

$$H_k\delta_v = f_k \qquad (18)$$

with

$$H_k = \mu I_N + \kappa_E + \text{div}\,\alpha D^{-1}(m^k)G\nabla$$

$$f_k = -\mu v^k + \text{div}\,\alpha D^{-1}(m^k)\nabla v^k + \kappa_E(-v^k + v^0)$$

Whenever H_k is not positive definite, we use the projection strategy presented by Hintermüller et al. (2003) to get a positive definite matrix which is used in place of H_k. See Lemma 3.4 in Hintermüller and Stadler (2006). We propose the following active set method for inpainting:

Algorithm: Primal-Dual Active Set (PDAS_I)

1. Set $k = 0$ and initialize $(v^0, p^0) \in \mathbb{R}^N \times \mathbb{R}^{2N}$.
2. Determine $\chi_{\mathcal{A}_{k+1}} \in \mathbb{R}^{2N\times 2N}$.
3. Compute H_k^+ if p^k is not feasible for all $i = 1,\ldots,N$. Otherwise set $H_k^+ = H_k$.
4. Solve for δ_v in $H_k^+\delta_v = f_k$ and compute δ_p.
5. Update $v^{k+1} = v^k + \delta_v$ and $p^{k+1} = p^k + \delta_p$.
6. Stop, or set $k := k + 1$ and go to step 2.

The algorithm above for inpainting is analogous to the semismooth method of Hintermüller and Stadler (2006) for denoising, with the same convergence properties as described in Theorem 3.6 of their paper. This algorithm can be used in simultaneous inpainting and removal of white noise.

Numerical Examples

The algorithm is implemented in MATLAB R2010b on a machine with a speed of 2.93 GHz and with 2 GB of RAM. Our test images are square grayscale images with dynamic range [0,255]. For strictly inpainting, the images are blur-free and noise-free, degraded only by thin lines and text which are the inpainting domains. The goal of inpainting is to reconstruct the inpainting domain by using the image information surrounding these domains. We compare the results of the proposed active set method with those of the split Bregman method (see, Cai et al. (2009), Goldstein and Osher (2009)). We are indebted to Pascal Getreuer (2010) for the split Bregman program codes to solve the discretized problem (2).

The method is terminated once the KKT residual, KKT_{res}, is significantly small. KKT_{res} is defined as

$$KKT_{res} = (KKT_1^2 + KKT_2^2)^{\frac{1}{2}}$$

where KKT_1 and KKT_2 are the left sides of the optimality conditions (14) and (15), respectively. Our first image sample is a 256 × 256 image Figure 1A. The image masked with text is Figure 1B. The inpainting domain is about 2 pixels wide. The mask is user-defined and is created using an image editing software. In this example, the parameter values for the algorithm are $\alpha = 10^{-5}$, $\mu = 10^{-9}$, $\gamma = 10^{-2}$, and $tol = 10^{-3}$. The small value of α is in consonance with the noise-free property of the image, lessening the smoothing effect of the TV term. Note further that the value of μ is taken to be significantly small compared to the other parameters, effectively rendering the solution independent of the regularizing term $||v||^2$. In all our experiments, the initial solution is the observed image.

The result obtained using the active set method is shown in Figure 1C. The method converged in 1 iteration, with a time of 3.7284 seconds. Split Bregman returned Figure 1D after 29 iterations in 1.8564 seconds. We observe that the proposed method gave a better reconstruction. For instance, residue of the text in the torus shape remains in the split Bregman result, whereas the PDAS_I result fully inpainted the region.

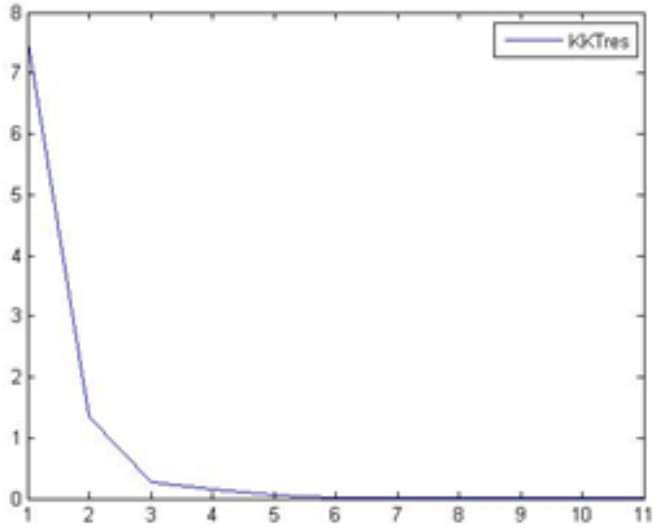

Figure 6. KTT residual.

Our second example is Figure 2A. We used the same parameters as in the first example. The reconstructions in Figure 3 show that the inpainting done by PDAS_I compares well with that of the split Bregman. Although the algorithm used 2.3556 seconds to converge, it did so in just one iteration, while the split Bregman converged in 12 iterations in 0.6240 seconds.

The third image example for inpainting, Figure 4B, has more thick inpainting domains than the domains in the previous examples. Here, parameter values are $\alpha = 10^{-3}$, $\mu = 10^{-7}$. The PDAS_I reconstruction is given in Figure 4C obtained in 5 iterations and 7.1916 seconds. The split Bregman converged in 87 iterations and a time of 2.9328 seconds. Comparing the reconstructions, we observe that the PDAS_I inpainted better than the split Bregman, e.g., on the line that comes across the legs of the tripod and along the edge of the coat. When the tolerance is reduced to $tol = 10^{-6}$, the method converged in 10 iterations after 15.2724 seconds.

The proposed method show promise in simultaneous inpainting and denoising. We add Gaussian noise to the clean image and the resulting noisy image is Figure 5A. The inpainting region is the same as in the last example. The noise level is 10%, $\alpha = 0.1$, and $tol = 10^{-6}$. The active set parameter $\gamma = 10^{-2}$. The result of the method is shown in Figure 5B. The reconstructed image is obtained in 10 iterations and 13.8374 seconds. The plot of KKT_{res} is in Figure 6.

We conclude the numerical tests with a comparison of the reconstructions made by the proposed method with the active set method (ASM) in Neri and Zara 2012. In the ASM, the TV model has $\frac{\mu}{2}\int_\Omega |\nabla u|^2$ as the regularizing item. Setting $\alpha = 0.05$, we varied the values of μ to see the effect of the regularizing terms on image reconstructions of Figure 7B. The results are in Figures 8 and 9. The ASM reconstructions tend to exhibit pronouncedly

Figure 7. A, Original image; **B**, Degraded image.

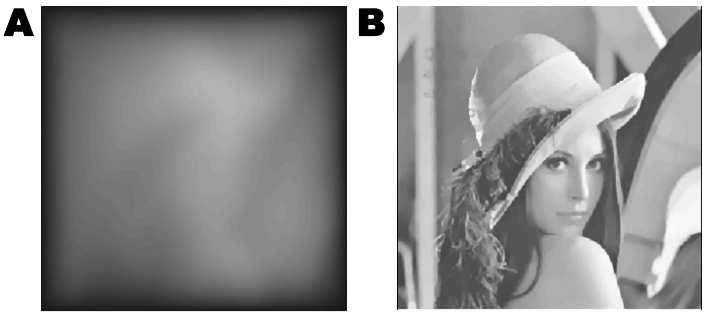

Figure 8. $\mu = 10^{-2}$. A, ASM; and **B**, PDAS_I.

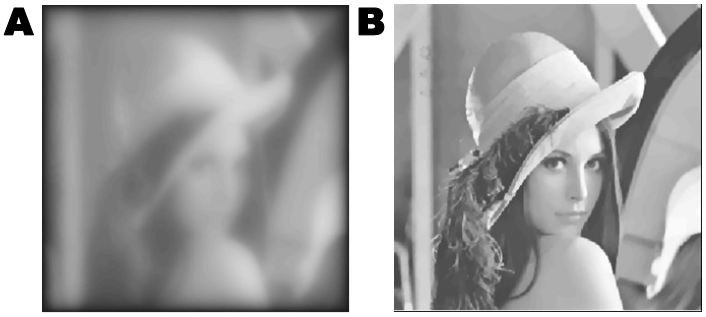

Figure 9. $\mu = 10^{-3}$. A, ASM; and **B**, PDAS_I.

the banding effect associated with TV models, while the PDAS_I reconstructions better preserve image features.

In all our test runs, we did not encounter any problem of ill-conditioning with H_k, for any k. Hence there was no need to modify the system matrix.

CONCLUSION

We presented here a regularized discrete variation model for simultaneous image inpainting and denoising. Further, we fitted a semismooth active set method to solve the problem. The method exploits the primal-dual structure of the model. Our numerical experiments show that the primal-dual active set method is very effective in providing inpainting reconstructions which are at least at par with recent methods such as the split Bregman. As

with most Newton-type methods, far fewer iterations are needed by PDAS_I to converge. The method also works well in removing low level Gaussian noise while inpainting.

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CONFLICTS OF INTEREST

There are no conflicts of interest arising from this research.

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